

Fast High-Accuracy Multi-Dimensional Pattern Inspection

Field of the Invention

This invention relates to machine vision, and particularly to systems for pattern inspection in an image.

1 Background of the Invention

2 Digital images are formed by many devices and used for many practical purposes. Devices
3 include TV cameras operating on visible or infrared light, line-scan sensors, flying spot scanners,
4 electron microscopes, X-ray devices including CT scanners, magnetic resonance imagers, and
5 other devices known to those skilled in the art. Practical applications are found in industrial
6 automation, medical diagnosis, satellite imaging for a variety of military, civilian, and scientific
7 purposes, photographic processing, surveillance and traffic monitoring, document processing,
8 and many others.

9 To serve these applications the images formed by the various devices are analyzed by digital
10 devices to extract appropriate information. One form of analysis that is of considerable practical
11 importance is determining the position, orientation, and size of patterns in an image that
12 correspond to objects in the field of view of the imaging device. Pattern location methods are of

1 particular importance in industrial automation, where they are used to guide robots and other
2 automation equipment in semiconductor manufacturing, electronics assembly, pharmaceuticals,
3 food processing, consumer goods manufacturing, and many others.

4 Another form of digital image analysis of practical importance is identifying differences between
5 an image of an object and a stored pattern that represents the “ideal” appearance of the object.
6 Methods for identifying these differences are generally referred to as pattern inspection methods,
7 and are used in industrial automation for assembly, packaging, quality control, and many other
8 purposes.

9 One early, widely-used method for pattern location and inspection is known as blob analysis. In
10 this method, the pixels of a digital image are classified as “object” or “background” by some
11 means, typically by comparing pixel gray-levels to a threshold. Pixels classified as object are
12 grouped into blobs using the rule that two object pixels are part of the same blob if they are
13 neighbors; this is known as connectivity analysis. For each such blob we determine properties
14 such as area, perimeter, center of mass, principal moments of inertia, and principal axes of
15 inertia. The position, orientation, and size of a blob is taken to be its center of mass, angle of first
16 principal axis of inertia, and area, respectively. These and the other blob properties can be
17 compared against a known ideal for purposes of inspection.

18 Blob analysis is relatively inexpensive to compute, allowing for fast operation on inexpensive
19 hardware. It is reasonably accurate under ideal conditions, and well-suited to objects whose
20 orientation and size are subject to change. One limitation is that accuracy can be severely
21 degraded if some of the object is missing or occluded, or if unexpected extra features are present.
22 Another limitation is that the values available for inspection purposes represent coarse features of

1 the object, and cannot be used to detect fine variations. The most severe limitation, however, is
2 that except under limited and well-controlled conditions there is in general no reliable method for
3 classifying pixels as object or background. These limitations forced developers to seek other
4 methods for pattern location and inspection.

5 Another method that achieved early widespread use is binary template matching. In this method,
6 a training image is used that contains an example of the pattern to be located. The subset of the
7 training image containing the example is thresholded to produce a binary pattern and then stored
8 in a memory. At run-time, images are presented that contain the object to be found. The stored
9 pattern is compared with like-sized subsets of the run-time image at all or selected positions, and
10 the position that best matches the stored pattern is considered the position of the object. Degree
11 of match at a given position of the pattern is simply the fraction of pattern pixels that match their
12 corresponding image pixel, thereby providing pattern inspection information.

13 Binary template matching does not depend on classifying image pixels as object or background,
14 and so it can be applied to a much wider variety of problems than blob analysis. It also is much
15 better able to tolerate missing or extra pattern features without severe loss of accuracy, and it is
16 able to detect finer differences between the pattern and the object. One limitation, however, is
17 that a binarization threshold is needed, which can be difficult to choose reliably in practice,
18 particularly under conditions of poor signal-to-noise ratio or when illumination intensity or
19 object contrast is subject to variation. Accuracy is typically limited to about one whole pixel due
20 to the substantial loss of information associated with thresholding. Even more serious, however,
21 is that binary template matching cannot measure object orientation and size. Furthermore,

1 accuracy degrades rapidly with small variations in orientation and/or size, and if larger variations
2 are expected the method cannot be used at all.

3 A significant improvement over binary template matching came with the advent of relatively
4 inexpensive methods for the use of gray-level normalized correlation for pattern location and
5 inspection. The methods are similar, except that no threshold is used so that the full range of
6 image gray-levels are considered, and the degree of match becomes the correlation coefficient
7 between the stored pattern and the image subset at a given position.

8 Since no binarization threshold is needed, and given the fundamental noise immunity of
9 correlation, performance is not significantly compromised under conditions of poor signal-to-
10 noise ratio or when illumination intensity or object contrast is subject to variation. Furthermore,
11 since there is no loss of information due to thresholding, position accuracy down to about $\frac{1}{4}$ pixel
12 is practical using well-known interpolation methods. The situation regarding orientation and size,
13 however, is not much improved with respect to binary template matching. Another limitation is
14 that in some applications, contrast can vary locally across an image of an object, resulting in poor
15 correlation with the stored pattern, and consequent failure to correctly locate it.

16 More recently, improvements to gray-level correlation have been developed that allow it to be
17 used in applications where significant variation in orientation and/or size is expected. In these
18 methods, the stored pattern is rotated and/or scaled by digital image re-sampling methods before
19 being matched against the image. By matching over a range of angles, sizes, and x-y positions,
20 one can locate an object in the corresponding multidimensional space. Note that such methods
21 would not work well with binary template matching, due the much more severe pixel
22 quantization errors associated with binary images.

1 One problem with these methods is the severe computational cost, both of digital re-sampling
2 and of searching a space with more than 2 dimensions. To manage this cost, the search methods
3 break up the problem into two or more phases. The earliest phase uses a coarse, subsampled
4 version of the pattern to cover the entire search space quickly and identify possible object
5 locations in the n-dimensional space. Subsequent phases use finer versions of the pattern to
6 refine the locations determined at earlier phases, and eliminate locations that the finer resolution
7 reveals are not well correlated with the pattern. Note that variations of these coarse-fine methods
8 have also been used with binary template matching and the original 2-dimensional correlation,
9 but are even more important with the higher-dimensional search space.

10 The location accuracy of these methods is limited both by how finely the multidimensional space
11 is searched, and by the ability of the discrete pixel grid to represent small changes in position,
12 orientation, and scale. The fineness of the search can be chosen to suit a given application, but
13 computational cost grows so rapidly with resolution and number of dimensions that practical
14 applications often cannot tolerate the cost or time needed to achieve high accuracy. The
15 limitations of the discrete pixel grid are more fundamental — no matter how finely the space is
16 searched, for typical patterns one cannot expect position accuracy to be much better than about $\frac{1}{4}$
17 pixel, orientation better than a degree or so, and scale better than a percent or so.

18 A similar situation holds when gray-level pixel-grid-based methods are used for pattern
19 inspection. Once the object has been located in the multidimensional space, pixels in the pattern
20 can be compared to each corresponding pixel in the image to identify differences. Some
21 differences, however, will result from the re-sampling process itself, because again the pixel grid
22 cannot accurately represent small variations in orientation and scale. These differences are

1 particularly severe in regions where image gray levels are changing rapidly, such as along object
2 boundaries. Often these are the most important regions of an object to inspect. Since in general,
3 differences due to re-sampling cannot be distinguished from those due to object defects,
4 inspection performance is compromised.

5

6 **Summary of the Invention**

7 In one general aspect, the invention is a method and apparatus for identifying differences
8 between a stored pattern and a matching image subset, where variations in pattern position,
9 orientation, and size do not give rise to false differences. The process of identifying differences is
10 called *inspection*. Generally, an object image must be precisely located prior to inspection. In
11 another general aspect, the invention is a system for analyzing an object image with respect to a
12 model pattern, wherein the system includes extracting pattern features from the model pattern;
13 generating a vector-valued function using the pattern features to provide a pattern field;
14 extracting image features from the object image; evaluating each image feature, using the pattern
15 field and an n-dimensional transformation that associates image features with pattern features, so
16 as to determine at least one associated feature characteristic; and using at least one feature
17 characteristic to identify at least one flaw in the object image. In a preferred embodiment, at least
18 one associated feature characteristic includes a probability value that indicates the likelihood that
19 an associated image feature does not correspond to a feature in the model pattern. In an alternate
20 preferred embodiment, the at least one associated feature characteristic includes a probability
21 value that indicates the likelihood that an associated image feature does correspond to a feature in
22 the model pattern.

1 In another preferred embodiment, at least one pattern feature includes a probability value
2 indicating the likelihood that the pattern feature does not correspond to at least one feature in the
3 object image.

4 When using at least one feature characteristic, it is preferred to transfer a feature
5 characteristic from the at least one image feature to an element of the pattern field, where in a
6 preferred embodiment, the element of the pattern field is the nearest element of the pattern field.

7 When using at least one feature characteristic, it is also preferred to use a plurality of
8 image features; and transfer a plurality of the feature characteristics from the plurality of image
9 features to a plurality of elements of the pattern field, wherein some of the plurality of elements
10 of the pattern field can include at least one link to a neighboring element of the pattern field.
11 Further, after transferring a plurality of the feature characteristics from the plurality of image
12 features to a plurality of elements of the pattern field, it is preferred that each element of the
13 plurality of elements of the pattern field receive a feature characteristic equal to the maximum of
14 its own feature characteristic and the feature characteristic of each neighboring element of the
15 pattern field.

16 In another preferred embodiment, to use at least one feature characteristic includes
17 identifying the nearest element of the pattern field; transferring a feature characteristic from the
18 at least one image feature to the nearest element of the pattern field; and computing a coverage
19 value using at least the transferred feature characteristic.

20 In a further preferred embodiment, evaluating each image feature includes comparing the
21 direction of each image feature with the direction of an element of the pattern field. It is
22 preferable to assign a higher weight to the image feature if the difference in the direction of the
23 image feature from the direction of an element of the pattern field is less than a specified

1 direction parameter. The specified direction parameter can be determined by a characteristic of
2 the element of the pattern field, such as a flag indicating "corner" or "non-corner".

3 It is also possible for a lower weight to be assigned to the image feature if the difference
4 in the direction of the image feature from the direction of an element of the pattern field is greater
5 than a specified direction parameter, wherein the specified direction parameter can be determined
6 by a characteristic of the element of the pattern field, such as a flag indicating "corner" or "non-
7 corner".

8 In yet another preferred embodiment, to evaluate each image feature includes comparing,
9 modulo 180 degrees, the direction of each image feature with the direction of an element of the
10 pattern field.

11 It is also possible that to evaluate each image feature includes assigning a weight of zero
12 when the image feature is at a position that corresponds to an element of the pattern field that
13 specified that no image feature is expected at that position.

14 Moreover, to evaluate each image feature can include comparing the distance of each
15 image feature with a specified distance parameter, where a lower weight can be assigned to the
16 image feature if the distance of the image feature is greater than a specified distance parameter,
17 or alternatively, where a higher weight can be assigned to the image feature if the distance of the
18 image feature is less than a specified distance parameter.

19 In another preferred embodiment, to evaluate each image feature includes comparing the
20 direction of each image feature with the direction of an element of the pattern field, and
21 comparing the distance of each image feature with a specified distance parameter.

22 To avoid ambiguity we will call the location of a pattern in a multidimensional space its
23 *pose*. More precisely, a pose is a coordinate transform that maps points in an image to

1 corresponding points in a stored pattern. In a preferred embodiment, a pose is a general six
2 degree of freedom linear coordinate transform. The six degrees of freedom can be represented by
3 the four elements of a 2x2 matrix, plus the two elements of a vector corresponding to the two
4 translational degrees of freedom. Alternatively and equivalently, the four non-translational
5 degrees of freedom can be represented in other ways, such as orientation, scale, aspect ratio, and
6 skew, or x-scale, y-scale, x-axis-angle, and y-axis-angle.

7 The invention can serve as a replacement for the fine resolution phase of any coarse-fine
8 method for pattern inspection, such as the prior art method of correlation search followed by
9 Golden Template Analysis. In combination with the coarse location phases of any such method,
10 the invention results in an overall method for pattern inspection that is faster and more accurate
11 than any known prior art method.

12 In a preferred embodiment, the PatQuick™ tool, sold by Cognex Corporation, Natick
13 MA, is used for producing an approximate object pose.

14 The invention uses a stored pattern that represents an ideal example of the object to be
15 found and inspected. The pattern can be created from a training image or synthesized from a
16 geometric description. According to the invention, patterns and images are represented by a
17 feature-based description that can be translated, rotated, and scaled to arbitrary precision much
18 faster than digital image re-sampling, and without pixel grid quantization errors. Thus accuracy is
19 not limited by the ability of a grid to represent small changes in position, orientation, or size (or
20 other degrees of freedom). Furthermore, pixel quantization errors due to digital re-sampling will
21 not cause false differences between the pattern and image that can limit inspection performance,
22 since no re-sampling is done.

1 Accuracy is also not limited by the fineness with which the space is searched, because the
2 invention does not test discrete positions within the space to determine the pose with the highest
3 degree of match. Instead the invention determines an accurate object pose from an approximate
4 starting pose in a small, fixed number of increments that is independent of the number of
5 dimensions of the space (i.e. degrees of freedom) and independent of the distance between the
6 starting and final poses, as long as the starting pose is within some "capture range" of the true
7 pose. Thus one does not need to sacrifice accuracy in order to keep execution time within the
8 bounds allowed by practical applications.

9 Unlike prior art methods where execution time grows rapidly with number of degrees of
10 freedom, with the method of the invention execution time grows at worst very slowly, and in
11 some embodiments not at all. Thus one need not sacrifice degree of freedom measurements in
12 order to keep execution time within practical bounds. Furthermore, allowing four or more
13 degrees of freedom to be refined will often result in better matches between the pattern and
14 image, and thereby improved accuracy.

15 The invention processes images with a feature detector to generate a description that is
16 not tied to a pixel grid. The description is a list of elements called *dipoles* (also called features)
17 that represent points (positions) along object boundaries. A dipole includes the coordinates of the
18 position of a point along an object boundary and a direction pointing substantially normal to the
19 boundary at that point. Object boundaries are defined as places where image gradient (a vector
20 describing rate and direction of gray-level change at each point in an image) reaches a local
21 maximum. In a preferred embodiment, gradient is estimated at an adjustable spatial resolution. In
22 another preferred embodiment, the dipole direction is the gradient direction. In another preferred

1 embodiment, a dipole, i.e., a feature, contains additional information as further described in the
2 drawings. In yet another preferred embodiment, dipoles are generated not from an image but
3 from a geometric description of an object, such as might be found in a CAD system.

4 The stored model pattern to be used by the invention for localization and subsequent
5 inspection is the basis for generating a dipole list that describes the objects to be found by
6 representing object boundaries. The dipole list derived from the model pattern is called the *field*
7 *dipole list*. It can be generated from a model training image containing an example object using a
8 feature detector, or it can be synthesized from a geometric description. The field dipole list is
9 used to generate a 2-dimensional vector-valued function called a *field*. For each point within the
10 region of the stored model pattern , the field gives a vector that indicates the distance and
11 direction to the nearest point along a model object boundary. The vector is called the *force* at the
12 specified point within the stored model pattern.

13 Note that the nearest point along a model object boundary is not necessarily one of the
14 model object boundary points represented by the field dipoles, but in general may lie between
15 field dipole positions. Note further that the point within the stored model pattern is not
16 necessarily an integer grid position, but is in general a real-valued position, known to within the
17 limits of precision of the apparatus used to perform the calculations. Note that since the force
18 vector points to the nearest boundary point, it must be normal to the boundary (except at
19 discontinuities).

20 In a preferred embodiment, if no model object boundary point lies within a certain range
21 of a field position, then a special code is given instead of a force vector. In another preferred
22 embodiment, the identity of the nearest field dipole is given in addition to the force. In another

1 preferred embodiment, one additional bit of information is given that indicates whether the
2 gradient direction at the boundary pointed to by the force is the same or 180° opposite from the
3 force direction (both are normal to the boundary). In another preferred embodiment, additional
4 information is given as further described in the drawings. In another embodiment, the field takes
5 a direction in addition to a position within the pattern, and the force returned is the distance and
6 direction to the nearest model object boundary point in approximately the given direction.

7 The stored model pattern used by the invention includes the field dipole list, the field, and
8 a set of operating parameters as appropriate to a given embodiment, and further described
9 throughout the specification.

10 Given an object image and an approximate starting pose, pattern localization proceeds as
11 follows. The object image is processed by a feature detector to produce a dipole list, called the
12 *image dipole list*. The starting pose is refined in a sequence of incremental improvements called
13 *attraction steps*. Each such step results in a significantly more accurate pose in all of the degrees
14 of freedom that are allowed to vary. The sequence can be terminated after a fixed number of
15 steps, and/or when no significant change in pose results from the last step, or based on any
16 reasonable criteria. In a preferred embodiment, the sequence is terminated after four steps.

17 For each attraction step, the image dipoles are processed in any convenient order. The
18 position and direction of each image dipole is mapped by the current pose transformation to
19 convert image coordinates to model pattern (field) coordinates. The field is used to determine the
20 force at the point to which the image dipole was mapped. Since each image dipole is presumed to
21 be located on an object boundary, and the force gives the distance and direction to the nearest
22 model object boundary of the stored model pattern, the existence of the image dipole at the

1 mapped position is taken as evidence that the pose should be modified so that the image dipole
2 moves in the force direction by an amount equal to the force distance.

3 It is important to note that object boundaries generally provide position information in a
4 direction normal to the boundary, which as noted above is the force direction, but no information
5 in a direction along the boundary. Thus the evidence provided by an image dipole constrains a
6 single degree of freedom only, specifically position along the line of force, and provides no
7 evidence in the direction normal to the force.

8 If the current pose is a fair approximation to the true object position, then many image
9 dipoles will provide good evidence as to how the pose should be modified to bring the image
10 boundaries into maximum agreement with the boundaries of the stored model pattern. For a
11 variety of reasons, however, many other image dipoles may provide false or misleading evidence.
12 Thus, it is important to evaluate the evidence provided by each image dipole, and assign a
13 weighting factor to each image dipole to indicate the relative reliability of the evidence.

14 In one embodiment, the direction (as mapped to the pattern coordinate system) of an
15 image dipole is compared with the force direction, and the result, modulo 180° , is used to
16 determine the weight of the image dipole. If the directions agree to within some specified
17 parameter, the dipole is given a high weight; if they disagree beyond some other specified
18 parameter, the dipole is given zero weight; if the direction difference falls between the two
19 parameters, intermediate weights are assigned.

20 In another embodiment, the image dipole direction is compared to the gradient direction
21 of the model pattern boundary to which the force points. A parameter is used to choose between

1 making the comparison modulo 180° , in which case gradient polarity is effectively ignored, or
2 making it modulo 360° , in which case gradient polarity is considered. In a preferred embodiment,
3 the field itself indicates at each point within the stored model pattern whether to ignore polarity,
4 consider polarity, or defer the decision to a global parameter.

5 In one embodiment, the force distance is used to determine the dipole weight. In a
6 preferred embodiment, if the force distance is larger than some specified parameter, the dipole is
7 given zero weight, on the assumption that the dipole is too far away to represent valid evidence.
8 If the force distance is smaller than some other specified parameter, the dipole is given a high
9 weight, and if it falls between the two parameters, intermediate weights are assigned.

10 In a preferred embodiment, the parameters specifying the weight factor as a function of
11 force distance are adjusted for each attraction step to account for the fact that the pose is
12 becoming more accurate, and therefore that one should expect image dipoles representing valid
13 evidence to be closer to the pattern boundaries.

14 In one embodiment, the gradient magnitude of the image dipoles is used to determine the
15 dipole weight. In a preferred embodiment, a combination of dipole direction, force distance, and
16 gradient magnitude is used to determine the weight.

17 For each attraction step, the invention determines a new pose that best accounts for the
18 evidence contributed by each image dipole, and taking into account the dipole's weight. In a
19 preferred embodiment, a least-squares method is used to determine the new pose.

20 The evaluation of each image dipole to produce a weight can also provide information for
21 inspection purposes. It is desirable to look for two distinct kinds of errors: missing features,

1 which are pattern features for which no corresponding image feature can be found, and extra
2 features, sometimes called "clutter", which are image features that correspond to no pattern
3 feature. In one embodiment, image dipoles with low weights are considered to be clutter. In a
4 preferred embodiment, a specific clutter value is computed for each image dipole, as further
5 described in the drawings below.

6 In an embodiment of the invention that can identify missing pattern features, the field at
7 each point gives identity of the nearest field dipole, if any, in addition to the force vector. Each
8 field dipole contains an evaluation, which is initialized to zero. Each image dipole transfers its
9 evaluation (also called "weight" or "feature characteristic") to that of the nearest field dipole as
10 indicated by the field. Since in general the correspondence between image and field dipoles is not
11 one-to-one, some field dipoles may receive evaluations (feature characteristics) from more than
12 one image dipole, and others may receive evaluations from none. Those field dipoles that receive
13 no evaluation may represent truly missing features, or may simply represent gaps in the transfer
14 due to quantization effects.

15 When more than one evaluation is transferred to a given field dipole, the evaluations can
16 be combined by any reasonable means. In a preferred embodiment, the largest such evaluation is
17 used and the others are discarded. Gaps in the transfer can be closed by considering neighboring
18 field dipoles. In one embodiment, methods known in the art as gray-level mathematical
19 morphology are used to close the gaps. In the case of the invention, one-dimensional versions of
20 morphological operations are used, since field dipoles lie along one-dimensional boundaries. In a
21 preferred embodiment, a morphological dilation operation is used.

1 If the starting pose is too far away from the true pose, there may be insufficient good evidence
2 from the image dipoles to move the pose in the right direction. The set of starting poses that
3 result in attraction to the true pose defines the *capture range* of the pattern. The capture range
4 depends on the specific pattern in use, and determines the accuracy needed from whatever
5 method is used to determine the starting pose.

6 In a preferred embodiment, the feature detector that is used to generate dipoles is tunable over a
7 wide range of spatial frequencies. If the feature detector is set to detect very fine features at a
8 relatively high resolution, the accuracy will be high but the capture range will be relatively small.
9 If on the other hand the feature detector is set to detect coarse features at a relatively lower
10 resolution, the accuracy will be lower but the capture range will be relatively large. This suggests
11 a multi-resolution method where a coarse, low resolution step is followed by a fine, high
12 resolution step. With this method, the capture range is determined by the coarse step and is
13 relatively large, while the accuracy is determined by the fine step and is high.

14

15 **Brief Description of the Drawing**

16 The invention will be more fully understood from the following detailed description, in
17 conjunction with the following figures, wherein:

18 Fig. 1 is a high-level block diagram of an embodiment of the invention;

19 Fig. 1A is an illustration of a pixel array having a 2:1 aspect ratio;

1 Fig. 2 is a block diagram of the training block of Fig. 1;

2 Fig. 3 is a block diagram of the feature detection block of Fig. 2;

3 Fig. 4 is a diagram showing bit assignments of a 32-bit word, and optional 'nearest

4 dipole' bits;

5 Fig. 5 is an illustration of a field element array, including a border of field elements;

6 Fig. 6 illustrates field seeding, showing some of the field elements of Fig. 5, including a

7 plurality of straight line segments of a pattern boundary, and the associated field dipoles;

8 Fig. 7A illustrates field dipole connecting, showing some of the field elements of Fig. 6,

9 including a plurality of straight line segments of a pattern boundary, and a plurality of associated

10 right and left links;

11 Figs. 7B and 7C are diagrams illustrating the order in which neighboring field elements

12 are examined;

13 Fig. 8 illustrates chain segmentation of Fig. 2, showing some of the field elements of Fig.

14 5, including a plurality of straight line segments, and a plurality of left and right links;

15 Figs. 9A, 9B, and 9C illustrate part of the analysis that is performed by the propagate

16 phase of the field generation module of Fig. 2;

17 Fig. 10 shows further details of the propagate phase of the field generation module of Fig.

18 2;

1 Fig. 11 shows the same portion of the field array that was shown after seeding in Fig. 6,
2 but with new force vectors resulting from one propagation;

3 Fig. 12 shows the same portion of the field array as Fig. 11, after two propagations;

4 Fig. 13 is a block diagram of the run-time module of the preferred embodiment of Fig. 1;

5 Fig. 14 is a diagram illustrating a least squares method for determining a pose that best
6 accounts for the evidence of the image dipoles at each attraction step;

7 Fig. 15 is a block diagram of the attraction module of Fig. 13;

8 Fig. 16 is a block diagram of the map module of Fig. 15;

9 Fig. 17 is a block diagram of the field module of Fig. 15;

10 Fig. 18 is a block diagram of the rotate module of Fig. 15;

11 Figs 19A, 19B, and 19C show output as a function of input for three fuzzy logic
12 processing elements;

13 Fig. 20A is a schematic diagram of the portion of the evaluate module of Fig. 15, showing
14 a preferred system for calculating 'weight' and 'eval';

15 Fig. 20B is a schematic diagram of the portion of the evaluate module of Fig. 15, showing
16 a preferred system for calculating 'clutter';

17 Fig. 21 is a schematic diagram of the sum module of Fig. 15;

1 Figs. 22A-D are block diagram of the solve module of Fig. 15, showing the equations and
2 inputs for providing 'motion' and 'rms error' for various degrees of freedom ;

3 Fig. 23 is a block diagram of the equations of the compose module of Fig. 15;

4 Fig. 24 is a block diagram of the equations of the Normal Tensor module of Fig. 15;

5 Fig. 25 is a graphical illustration of a plurality of image dipoles and a plurality of
6 connected field dipoles, showing field dipole evaluation;

7 Fig. 26 is a high-level block diagram of a multi-resolution embodiment of the invention;

8 Figs. 27A and 27B are flow diagrams illustrating the sequence of operations performed
9 by the modules of Fig. 15; and

10 Fig. 28 is a flow diagram illustrating a multi-resolution mode of operation of the
11 invention.

12

13

14 **Detailed Description of the Preferred Embodiments**

15 In the following figures, "modules" can be implemented as software, firmware, or hardware.

16 Moreover, each module may include sub-modules, or "steps", each of which can be implemented
17 as either hardware, software, or some combination thereof. Figure 1 is a high-level block
18 diagram of one embodiment of the invention. A training (model) image **100** containing an
19 example of a pattern **105** to be used for localization and/or inspection is presented. A training

1 module 110 analyzes the training image and produces a stored model pattern 120 for subsequent
2 use. At least one run-time image 130 is presented, each such image containing zero or more
3 instances of patterns 135 similar in shape, but possibly different in size and orientation, to the
4 training (model) pattern 105.

5 Each run-time image 130 has an associated *client map* 131, chosen by a user for a particular
6 application. A client map is a coordinate transformation that maps, i.e., associates points in an
7 orthonormal but otherwise arbitrary coordinate system to points in the image. An orthonormal
8 coordinate system has perpendicular axes, each axis having a unit scale. The client map provides
9 an orthonormal reference that is necessary to properly handle the orientation degree of freedom,
10 as well as the skew, scale, and aspect ratio degrees of freedom. In practical applications, the
11 images themselves are almost never orthonormal, since practical image sensors almost never
12 have perfectly square pixels. For example, pixels having an aspect ration of 2:1 are possible, as
13 shown in Fig. 1A. In this case, the client map would be a 2x2 matrix:

14

15 0.5 0.0

16 0.0 1.0

17

18 If the pixels are square, the client map is the identity transform, i.e., each diagonal entry in the
19 transform matrix is 1.0, and each off-diagonal element is 0.0.

1 Furthermore, it is sometimes useful to have a significantly non-orthonormal field. For example, a
2 field generated from a square pattern can be used to localize and inspect a rectangular or even
3 parallelogram-shaped instance of the pattern by using an appropriate starting pose. These cases
4 can only be handled if an orthonormal reference is available.

5 For each run-time image, a starting pose **132** is determined by any suitable method, such as
6 coarse gray-level correlation with orientation and size re-sampling, a coarse generalized Hough
7 transform, or the Cognex PatQuick™ tool. The starting pose **132** is a six-degree-of-freedom
8 coordinate transformation that maps points in the pattern **105** to approximately corresponding
9 points **135** in the run-time image **130**. A run-time module **140** analyzes the image **130**, using the
10 stored pattern **120**, the starting pose **132**, and the client map **131**. As a result of the analysis, the
11 run-time module **140** produces a pose **134** that maps pattern points **105** to accurately
12 corresponding image points **135**.

13 The run-time module **140** produces an *rms error* value **136** that is a measure of the degree of
14 match between the pattern **105** and the image **130**. The rms error value **136** is the root-mean-
15 square error from the least squares solution, or other error minimization solution, that is used to
16 determine a pose that best fits the evidence of the image dipoles, to be described in more detail
17 below. A value of zero represents a perfect fit, while higher values represent poorer fits.

18 The run-time module **140** produces a *coverage* value **138** that is a measure of the fraction of the
19 pattern **105** to which corresponding image features have been found. The coverage value **138** is
20 computed by summing the field dipole evaluations and dividing by the number of field dipoles,
21 to be described in more detail below.

1 The run-time module **140** produces a *clutter* value that is a measure of extra features found in the
2 image that do not correspond to pattern features. In a preferred embodiment, the clutter value is
3 computed by summing the individual image dipole clutter values and dividing by the number of
4 field dipoles.

5 The run-time module **140** produces an evaluated image dipole list **150** and an evaluated field
6 dipole list **160**. The evaluated image dipole list 150 identifies features in the image **130** not
7 present in the pattern **105**, and the evaluated field dipole list 160 identifies features in the pattern
8 **105** not present in the image **130**. The differences between the image and pattern so identified
9 can be used for inspection purposes.

10 Figure 2 shows a block diagram of the training module 110 and pattern-related data 120. A
11 training image **100** containing an example of a pattern 105 to be localized and/or inspected is
12 presented to training module **110**, which analyzes the training image 100 and produces a stored
13 pattern **120** for subsequent use. The training module **110** consists of two modules, a feature
14 detection module **200** and a field generation module **210**. Both modules use various parameters
15 **220** to control operation as appropriate for the application. These parameters, as well as those
16 needed for subsequent run-time modules, are pattern-dependent and therefore are collected and
17 stored in the pattern **120** as shown.

18 The feature detection module **200** processes the training image **100**, and using the parameters
19 **220**, to produce a field dipole set **230**, which is stored in the pattern **120** as shown. The field
20 generation module **210** uses the field dipole set 230 and parameters 220 to produce a field **240**,
21 stored in the pattern **120** as shown.

1 As described in the summary above, the field 240 produces information as a function of
2 theoretically real-valued position within the region of the pattern 105. In practice, since all the
3 field values cannot be computed analytically and stored, a 2-dimensional array is used that stores
4 field values at discrete points on a grid. Given a real-valued position (to some precision
5 determined by the particular details of the embodiment), an interpolation method, such as the
6 method shown in Fig. 17, is used to compute field values at intermediate points between grid
7 elements. Since the field grid is never translated, rotated, or scaled, no re-sampling is needed and
8 grid quantization effects are small. Instead, the image dipoles, which are not grid-based, are
9 mapped to the fixed field coordinates in accordance with the map of Fig. 16.

10 Thus the purpose of the field generation module 210 is to compute the elements of the 2-
11 dimensional array that encodes the field 240. The field generation module 210 is itself composed
12 of many steps or submodules, as shown in figure 2. Each of these steps modifies the field in
13 some way, generally based on the results of the previous steps. Some of the steps also add
14 information to the field dipole set 230. The specific sequence of steps shown in figure 2
15 corresponds to a preferred embodiment; many other variations are possible within the spirit of
16 the invention; the essential requirement is that the stored pattern 120 be able to provide certain
17 information as a function of position within the region of the pattern 105.

18 In a preferred embodiment as shown in figure 2, the field generation module 210 consists of the
19 following steps. An initialization step 250 loads predefined codes into the field array elements
20 520, 540, shown in Fig. 5. A seed step 252 sets up field array elements at positions corresponding
21 to the dipoles in the field dipole set 230. A connect step 254 uses the seeded field array to
22 identify neighboring dipoles for each field dipole. Once identified, the field dipoles are connected

1 to neighboring ones to form chains by storing the identity of left and right neighbors along
2 pattern boundaries, if any. A chain step **256** scans the connected field dipoles to identify and
3 catalog discrete chains. For each chain, the starting and ending dipoles, length, total gradient
4 magnitude, and whether the chain is open or closed is determined and stored.

5 A filter step **258** removes weak chains from the pattern by removing the dipoles they contain
6 from the field array (i.e. reversing the seeding step 252 for those dipoles). A variety of criteria
7 can be used to identify weak chains. In a preferred embodiment, chains whose total gradient
8 magnitude or average gradient magnitude are below some specified parameter are considered
9 weak.

10 A segment step **260** divides chains into segments of low curvature, separated by zones of high
11 curvature called *corners*. Corner dipoles are marked in the field array for use as described in
12 subsequent figures. Curvature can be determined by a variety of methods; in a preferred
13 embodiment, a dipole is considered a corner if its direction differs from that of either neighbor by
14 more than some specified parameter, e.g., as further described in conjunction with Fig. 8.

15 A sequence of zero or more propagate steps **262** extend the field out from the seeded positions.
16 The result is that force vectors pointing to pattern boundaries, as well as other information
17 needed by the run-time steps, can be obtained at some distance from the boundaries. The number
18 of propagate steps **262** is controlled by a parameter and determines the distance from pattern
19 boundaries that the field will contain valid force vectors, as well as the computation time needed
20 for pattern training. In a preferred embodiment, four propagation steps are used. Field elements
21 beyond the range of propagation will contain the code set during the initialization step **250**.

1 Figure 3 shows a preferred embodiment of a feature detector to be used for practice of the
2 invention. The feature detector processes a source image **300**, which can be either a training
3 image or a run-time image. A low-pass filter **310** and image sub-sampler **320** are used to
4 attenuate fine detail in the source image that for a variety of reasons we wish to ignore. For
5 example we may wish to attenuate noise or fine texture, or we may wish to expand the capture
6 range of the pattern by focusing on coarse pattern features. Also, we may wish to decrease the
7 number of image dipoles, thereby reducing processing time.

8 The response of the filter **310** and sub-sampler **320** are controlled by parameters **220** stored in the
9 pattern (not shown in this figure). One setting of the parameters effectively disables the filter and
10 sub-sampler, allowing the source image **300** to pass without modification for maximum
11 resolution.

12 Methods for low-pass filtering and sub-sampling of digital images are well known in the art. In a
13 preferred embodiment, a constant-time second-order approximately Gaussian filter is used, as
14 described in [U.S. patent pending "Efficient, Flexible Digital Filtering"].

15 The filtered, sub-sampled image is processed by a gradient estimation module **330** to produce an
16 estimate of the x (horizontal) and y (vertical) components of image gradient at each pixel. A
17 Cartesian-to-polar conversion module **340** converts the x and y components of gradient to
18 magnitude and direction. A peak detection module **350** identifies points where the gradient
19 magnitude exceeds a noise threshold and is a local maximum along a 1-dimensional profile that
20 lies in approximately the gradient direction, and produces a list of the grid coordinates (row and
21 column number), gradient magnitude, and gradient direction for each such point.

1 A sub-pixel interpolation module **360** interpolates the position of maximum gradient magnitude
2 along said 1-dimensional profile to determine real-valued (to some precision) coordinates (x_i, y_i)
3 of the point. The result is a list of points that lie along boundaries in the image, which includes
4 the coordinates, direction, and magnitude of each point. This list can be used as the basis for
5 either a field or image dipole list, to which additional information may be added as appropriate.

6 Methods for identifying points along image boundaries are well-known in the art. Any such
7 method can be used for practice of the invention, whether based on gradient estimation or other
8 techniques. Methods for gradient estimation, Cartesian-to-polar conversion, peak detection, and
9 interpolation are also well-known. In a preferred embodiment, the methods described in [U.S.
10 patent pending "Method and Apparatus for Fast, Inexpensive, Subpixel Edge Detection"] are
11 used.

12 In a preferred embodiment, the source image has eight bits of gray-scale per pixel. The low-pass
13 filter produces a 16-bit image, taking advantage of the inherent noise-reduction properties of a
14 low-pass filter. The gradient estimation module uses the well-known Sobel kernels and operates
15 on either a 16-bit filtered image, if the parameters are set so as to enable the filter 310, or an 8-bit
16 unfiltered image if the parameters are set so as to disable the filter 310. The x and y components
17 of gradient are always calculated to 16 bits to avoid loss of precision, and the gradient magnitude
18 and direction are calculated to at least six bits preferably using the well-known CORDIC
19 algorithm.

20 Several parameter values are needed for feature extraction, both in the training module **110** and
21 in the run-time module **140**. Generally these parameters include those controlling the response of
22 the low-pass filter **310**, the sub-sampling amount used by sub-sampler **320**, and the noise

1 threshold used by peak detector **350**. Other values may be needed depending on the exact details
2 of the feature extractor used to practice the invention.

3 Appropriate settings for the parameter values depend on the nature of the patterns and images to
4 be analyzed. In a preferred embodiment certain defaults are used that work well in many cases,
5 but in general no rules can be given that work well in all cases. Said preferred embodiment is
6 further described below in conjunction with figure 26.

7 Figure 4 shows an element of the field array as used in a preferred embodiment of the invention.
8 Information is packed into a 32-bit word **400**, both to conserve memory and to speed up access
9 on conventional computers by maximizing the number of elements that will fit in data cache and
10 using a word size that keeps all elements properly aligned on appropriate address boundaries.
11 Fixed point representations are used for the force vector, both because they are more compact
12 than floating point representations and to allow best use to be made of the signal processing
13 capabilities of modern processors such as the Texas Instruments TMS320C80 and the Intel
14 Pentium-MMX.

15 In a preferred embodiment, a field element stores a force vector that gives the distance and
16 direction to the nearest point along a pattern boundary, and one bit that specifies whether the
17 gradient direction at that boundary point is in the same, or 180° opposite, direction as the force
18 vector. This is accomplished as shown in figure 4 by storing a signed force magnitude **410** and a
19 gradient direction **420**. If the force direction is the same as the gradient direction, the force
20 magnitude **410** will be positive; if the force direction is opposite from the gradient direction, the
21 force magnitude **410** will be negative.

1 The magnitude/direction representation for the force vector is preferred over an x-y component
2 representation because it is necessary to be able to represent vectors that have zero length but a
3 well-defined direction. Such vectors are called *pseudo-null* vectors. The equivalent x-y
4 components can be calculated by the well-known formula

$$5 \quad \begin{pmatrix} force_x \\ force_y \end{pmatrix} = magnitude \begin{pmatrix} \cos(direction) \\ \sin(direction) \end{pmatrix}$$

6 Note that gradient direction can be used in the above formula, since the stored magnitude is
7 negative if the force direction is opposite the gradient direction.

8 In the preferred embodiment shown, the force magnitude **410** is in units of field grid increments,
9 using a two's complement representation of 16 total bits, of which the least significant 11 are to
10 the right of the binary point and the most significant is the sign bit. Thus the maximum force
11 vector length is just under 16 field grid units, and the resolution is $1/2048^{th}$ of a grid unit.

12 The gradient direction **420** is preferably represented as a 12-bit binary angle in the range 0° to
13 360° , with a resolution of $360^\circ/4096 = 0.088^\circ$. In other embodiments, the bits of field element
14 400 are divided between force 410 and direction 420 to provide greater or lesser precision and
15 range, as needed for each particular application.

16 A 4-bit flags element **430** is also stored in the field element **400**. An 2-bit eval code **440**
17 determines how an image dipole is to be evaluated if the current pose maps it to a field position
18 within the region covered by this element 400. The *don't care* code specifies that the image
19 dipole should be ignored. The *expect blank* code specifies that no features are expected in this
20 region of the pattern, and so if any image dipoles map here they should be given a low evaluation

1 for inspection purposes, should be given a high clutter rating, and should not be used as evidence
2 for localization. The *evaluate only* code specifies that the image dipole should be evaluated by
3 the usual criteria for inspection purposes, but should not contribute evidence for localization
4 purposes. The *attract* code specifies that the image dipole should be evaluated and used both for
5 localization and inspection.

6 If the eval code **440** is either “don’t care” or “expect blank”, the force vector is undefined and is
7 said to be invalid. If the eval code is “evaluate only” or “attract”, the force vector is said to be
8 valid.

9 A 1-bit corner code **450** specifies whether or not the pattern boundary point pointed to by the
10 force vector is in a high-curvature zone (“is corner”) or a low-curvature segment (“not corner”).

11 If the force vector is invalid, the corner code is set to “not corner”.

12 A 1-bit polarity code **460** specifies whether the image dipole evaluation should consider or ignore
13 gradient direction, as described above in the summary section and further described below. A
14 parameter is used to specify whether or not to override the polarity flags stored in the field
15 element 400, and if so, whether to force polarity to be considered or ignored.

16 In a preferred embodiment, the field element 400 also specifies the identity of the nearest field
17 dipole 401 in addition to the force vector 400. The identity 401 can be represented as a index into
18 the field dipole list. In a preferred embodiment, a 16-bit index is used, which is stored in a
19 separate array so as to satisfy data alignment guidelines of conventional computers.

20 Figure 5 shows details of the initialization step **250** of the field generation step **210**. A 2-
21 dimensional array **500** of field elements **400** is used. Any reasonable grid spacing can be used; in

1 a preferred embodiment, the grid spacing is the same as that of the image that is input to the
2 gradient estimation module **330** of the feature detector **200**.

3 Field elements **520**, (indicated as white in figure 5 and having the same structure as field element
4 **400**) cover the region of the training pattern **105**, together forming a “training region”. The field
5 elements **520** are initialized so that the eval code **440** is set to “expect blank”. As described
6 above, in this state the force vector is considered invalid and need not be initialized. In one
7 embodiment, however, further described below in conjunction with figure 20b, the gradient
8 direction field **420** of these field elements **520** are set equal to the corresponding gradient
9 directions of the training image. A border of additional field elements **540**, (indicated as gray in
10 figure 5 and having the same structure as field element **400**) are initialized so that the eval code
11 **440** is set to “don’t care”. This reflects the fact that in general we don’t know what features might
12 lie beyond the bounds of the training region. These “don’t care” values will be replicated inwards
13 during each propagation step **262**, so that image features lying just outside the training pattern
14 **105** don’t attract to pattern features just inside.

15 A separate corresponding array of field dipole indices **401**, identical in size to the white-shaded
16 field elements **520**, is also used, but need not be initialized. The values in this array are
17 considered valid only if the force vector of the corresponding field element of array **500** is valid.

18 Figure 6 shows details of the seed step **252** of the field generation module **210**. Shown is a subset
19 of the field elements **520** of the field array **500**. Each field dipole is located within some field
20 element. For example, the field dipole at point **600** falls within field element **620**, indicated as
21 gray in figure 6. Also shown is a small straight-line section of pattern boundary **660**

1 corresponding to the example field dipole at point 600. This section of boundary is shown
2 primarily to aid in understanding the figure. Its orientation, and position along a line normal to its
3 orientation, are significant, but its length is essentially arbitrary.

4 The field element 620 is set to have force vector 640. The force vector points from the center of
5 element 620 to a point on boundary section 660 and either in the direction, or opposite to the
6 direction of the dipole (i.e. normal to the boundary), whichever is required to bring the head of
7 the vector to the boundary 660. In the example shown, the point on the boundary to which the
8 vector points is coincident with the dipole position 600, but in general it need not be. Figure 6
9 shows several other examples of seeded force vectors.

10 It also may happen that a field dipole's position falls exactly at the center of a field element, so
11 that the length of the force vector is zero. In this case the force vector is pseudo-null — its
12 direction is well-defined and must be set properly.

13 In a preferred embodiment, for each field element that receives a seed force vector, the eval code
14 is set to "attract", the corner code is set to "no corner", and the polarity code is set to "consider
15 polarity". Other schemes may be devised within the spirit of the invention to suit specific
16 applications.

17 For each field element that receives a seed force vector, the corresponding element 401 of the
18 array of field dipole indices is set to identify the field dipole used to seed the field element.

19 In a preferred embodiment using the feature detector of figure 3, as further described in [U.S.
20 patent pending "Method and Apparatus for Fast, Inexpensive, Subpixel Edge Detection"], and
21 where the field grid has the same geometry as the image that is input to the gradient estimation

1 module **330**, no more than one field dipole will fall within any given field element, and there will
2 be no gaps in the boundary due to grid quantization effects. In a less preferred embodiment using
3 different methods for feature detection, various schemes can be used to handle multiple dipoles
4 that fall within a given field element, or gaps in the boundary due to quantization effects. The
5 preferred method for multiple dipoles within one field element is to choose the one whose force
6 vector is shortest, and discard the others. The preferred method for gaps in the boundary is to do
7 nothing and let the propagation steps fill in the gaps.

8 Figure 7 shows details of the connect step **254** of the field generation module **210**. Figure 7a
9 shows the same subset of field elements **520** of the field array **500** as was shown in figure 6. Also
10 shown is the example field element **620**, indicated as light gray.

11 For every field dipole, the seeded field is examined to identify neighboring positions that contain
12 dipoles to which the dipole should be connected. For the example field element **620**, the
13 neighboring positions **700** are shown, shaded medium gray. The neighboring positions **700** are
14 examined in two steps of four neighboring positions each, each step in a particular order,
15 determined by the direction of the field dipole corresponding to field element **620**.

16 In one step, a left neighbor field element **710** is identified, and a left link **715** is stored in the field
17 dipole corresponding to field element **620** identifying the field dipole corresponding to field
18 element **710** as its left neighbor. In the other step, a right neighbor field element **720** is identified,
19 and a right link **725** is stored to identify the field dipole's right neighbor. If a given neighbor
20 cannot be found, a null link is stored. Note that "left" and "right" are defined arbitrarily but
21 consistently by an imaginary observer looking along the dipole gradient direction.

1 Figure 7b shows the order in which neighboring field elements are examined for a dipole whose
2 direction falls between arrows **740** and **742**, corresponding to a pattern boundary that falls
3 between dotted lines **744** and **746**. The sequence for identifying the left neighbor is +1, +2, +3,
4 and +4. The first neighbor in said sequence that contains a dipole (seeded field element), if any,
5 is the left neighbor. Similarly, the sequence for identifying the right neighbor is -1, -2, -3, and -4.

6 Figure 7c shows another example, where the dipole direction falls between arrows **760** and **762**,
7 corresponding to a pattern boundary that falls between dotted lines **764** and **766**. The sequences
8 of neighbors are as shown. The sequences for all other dipole directions are simply rotations of
9 the two cases of figures 7b and 7c.

10 Note that the sequences given in figures 7b and 7c show a preference for orthogonal neighbors
11 over diagonal neighbors, even when diagonal neighbors are “closer” to the direction of the
12 pattern boundary. This preference insures that the chains will properly follow a stair-step pattern
13 for boundaries not aligned with the grid axes. Clearly this preference is somewhat dependent on
14 the specific details of how the feature detector chooses points along the boundary.

15 Once connections have been established for all field dipoles, a consistency check is performed.
16 Specifically, the right neighbor of a dipole's left neighbor should be the dipole itself, and the left
17 neighbor of a dipole's right neighbor should also be the dipole itself. If any links are found for
18 which these conditions do not hold, the links are broken by replacing them with a null link. At
19 the end of the connect step, only consistent chains remain.

1 Many alternate methods can be used to connect dipoles within the spirit and scope of the
2 invention. In some embodiments, particularly where no inspection is to be performed, the
3 connect 254 is omitted entirely.

4 Figure 8 shows details of the segment step **260** of the field generation step **210**. Field elements
5 **800** shaded medium gray are identified as “corners” because the dipole directions differs from
6 that of their left and/or right neighbors by more than some specified parameter. In a preferred
7 embodiment, the parameter is 16.875 degrees. For these elements the corner code **450** is set to
8 “is corner”. Other field elements **820**, shaded light gray, lie along one chain segment, while field
9 elements **840**, also shaded light gray, lie along another chain segment. For these elements it is not
10 necessary to set the corner code, because it was set to “no corner” when the field was seeded.

11 Figure 9 shows examples of part of the analysis that is performed by the propagate step **262** of
12 the field generation step **210**. In the example of figure 9a, field element **900** initially does not
13 have a valid force vector; its eval code is “expect blank”, as set by the initialization step **250**.
14 Neighboring element **902** has a valid force vector **904**, which points to a segment of pattern
15 boundary **906** that is assumed to be an approximately straight line.

16 A vector **908** is constructed from the center **916** of element **900** to the center of the neighbor **902**.
17 The projection **910** of vector **908** onto force vector **904** is constructed. A new force vector **912** is
18 constructed from the center **916** of field element **900** to the boundary **906** by adding the
19 neighbor’s force vector **904** to the projection **910**. An offset value is computed whose magnitude
20 is equal to the length **914** of the difference between vector **908** and projection **910**, and whose
21 sign is determined by the direction **918** by which vector **908** must be rotated to coincide with

1 projection **910**, where anti-clockwise is positive as shown and clockwise is negative. The result
2 of this analysis is the new force vector **912** and offset value of magnitude **914** and sign **918**.

3 A similar example but for a diagonal neighbor **932** of element **930** is shown in figure 9b. The
4 projection **940** of vector **938** onto force vector **934** is constructed. A new force vector **942** is
5 constructed from the center **946** of field element **930** to the boundary **936** by adding the
6 neighbor's force vector **934** to the projection **940**. An offset value is computed whose magnitude
7 is equal to the length **944** of the difference between vector **938** and projection **940**, and whose
8 sign is negative since vector **938** must be rotated clockwise **948** to coincide with projection **940**.

9 Another example is shown in figure 9c, where in this case the boundary **966** passes between field
10 element **960** and its neighbor **962**. The projection **970** of vector **968** onto force vector **964** is
11 constructed. A new force vector **972** is constructed from the center **976** of field element **960** to
12 the boundary **966** by adding the neighbor's force vector **964** to the projection **970**. An offset
13 value is computed whose magnitude is equal to the length **974** of the difference between vector
14 **968** and projection **970**, and whose sign is negative since vector **968** must be rotated clockwise
15 **978** to coincide with projection **970**.

16 Further details of propagate step **262** of the field generation step **210** are shown in figure 10.
17 Each element of the field array is examined. Any element whose eval code **440** is "expect blank"
18 is considered for possible propagation of the field to that element. All other field elements are
19 already in a final state and are skipped. For each field element so considered, the eight neighbors
20 are examined. If two or more adjacent neighbors have valid force vectors or have eval codes

1 equal to “don’t care”, the field will be propagated to the said field element; otherwise, the field
2 element will be skipped and possibly considered again on a subsequent propagate step.

3 The rule specifying two or more adjacent neighbors is used to insure that there is sufficient
4 information to be able to interpolate the field between neighbors. “Adjacent” means either
5 sharing an edge, such as elements **1010** and **1012** of figure 10, or sharing a corner, such as
6 elements **1010** and **1014**.

7 In figure 10 element **1000** shaded light gray has eval code “expect blank”, and neighbors **1010**,
8 **1012**, and **1014**, shaded medium gray, have valid force vectors (the field has been seeded at or
9 already propagated to the neighbors). Neighboring element **1010** has force vector **1030**, and
10 following the method of figure 9 new force vector **1032** and positive offset **1034** are computed.
11 Neighboring element **1012** has force vector **1040**, and following the method of figure 9 new force
12 vector **1042** and negative offset **1044** are computed. Neighboring element **1014** has force vector
13 **1050**, and following the method of figure 9 new force vector **1052** and negative offset **1054** are
14 computed.

15 The neighbors of field element **1000** are scanned anti-clockwise in sequence **1070**. The starting
16 and ending points of sequence **1070** are arbitrary. If exactly one positive to negative offset
17 transition between adjacent neighbors is found, the field is propagated to element **1000** by
18 constructing a force vector **1080** by interpolating between new force vectors **1032** and **1042**.

19 In a preferred embodiment, the interpolation is a weighted average of vectors **1032** and **1042**.

20 The vector **1032** is weighted by the magnitude of offset **1044**, and the vector **1042** is weighted by
21 the magnitude of offset **1034**. The effect is that the weight of a vector is proportional to the other

1 vector's offset and inversely proportional to its own offset, so that vectors are more heavily
2 weighted if they pass closer to their corresponding neighbor's force vector. As shown in figure
3 10, the offset corresponding to vector **1032** has the smaller magnitude, so it is more heavily
4 weighted and therefore force vector **1080** passes closer to vector **1032** than to vector **1042**.

5 One method for constructing a weighted average of vectors is to scale each vector by its
6 corresponding weight, add the results, and then scale by the inverse of the sum of the weights.
7 This is equivalent to an independent weighted average of the x and y components. In a preferred
8 embodiment, an independent weighted average of the magnitude and direction is used.

9 If the vectors **1032** and **1042** participating in the interpolation are pointing to distant points along
10 a pattern boundary, or to different boundaries, the field is considered indeterminate at element
11 **1000** and the eval code is set to "don't care". In a preferred embodiment, the vectors are
12 considered to be pointing to distant points or different boundaries if either their magnitudes differ
13 by more than 3 grid units or their directions differ by more than 135°.

14 In a preferred embodiment, a special case rule is used to propagate the field at the ends of an
15 open chain. If a neighboring element with a valid force produces a small positive offset, and no
16 anti-clockwise adjacent neighbor has a valid force, the field will propagate without interpolation
17 by using the new force vector as constructed by the method of figure 9. Similarly, if a
18 neighboring element with a valid force produces a small negative offset, and no clockwise
19 adjacent neighbor has a valid force, the field will propagate without interpolation by using the
20 new force vector as constructed by the method of figure 9. In a preferred embodiment, a small
21 offset is one whose magnitude is less than 1/10th of a grid unit.

1 If more than one positive to negative offset transition between adjacent neighbors, or application
2 of the special case rule, is found, or if none are found, the field is considered indeterminate at
3 element **1000** and the eval code is set to “don’t care”. One reason that no such transitions might
4 be found is that neighboring field elements are themselves set to “don’t care”, for example the
5 border elements **540** set by the initialization step **250**.

6 If a valid force is propagated to element **1000**, then corner code **450**, polarity code **460**, and the
7 index of the nearest field dipole are also propagated by copying from whichever of the
8 neighboring elements participating in the interpolation has the smallest offset magnitude (greatest
9 weight). In the example of figure 10, the values would be copied from element **1010**. If the
10 special case rule was applied, the values are copied from the neighbor with small offset used to
11 construct the new force vector.

12 Many variations on the above rules can be used within the spirit of the invention to achieve
13 similar results. Indeed any method that produces force vectors that point to the nearest point
14 along a pattern boundary can be used to practice this invention.

15 Figure 11 shows the same portion of the field array that was shown after seeding in figure 6, but
16 with new force vectors resulting from one propagation step. Figure 12 shows the same portion
17 after two propagation steps. Note in figure 12 field element **1200** whose eval code is set to “don’t
18 care” because more than one positive to negative offset transition between adjacent neighbors
19 was found.

20 Figure 13 shows a block diagram of the run-time module **140** of a preferred embodiment. Run-
21 time module **140** analyzes the image **130**, using the stored pattern **120**, the starting pose **132**, and

1 the client map 131. As a result of the analysis, the run-time module produces a pose 134 that
2 maps pattern points to accurately corresponding image points.

3 The run-time module **140** produces an *rms error* value 136 that is a measure of the degree of
4 match between the pattern and the image, a *coverage* value 138 that is a measure of the fraction
5 of the pattern to which corresponding image features have been found, and a clutter value 137
6 that is a measure of extra features found in the image that do not correspond to pattern features.

7 The run-time module **140** produces an evaluated image dipole list 150 and an evaluated field
8 dipole list **160**, e.g., as shown in Fig. 25. The clutter values of the evaluated image dipole list 150
9 can be used to identify features in the image **130** not present in the pattern **105** (shown in figure
10 1). These probability values range from 0 to 1 and indicate the likelihood that the image feature
11 is not present in the pattern. The “eval2” values (Fig. 25) of the evaluated field dipole list 160
12 can be used to identify features in the pattern **105** not present in the image **130**. These
13 probability values range from 0 to 1, and indicate the likelihood that the pattern feature was
14 found in the image.

15 The run-time module 140 uses a feature detection module **200** to process the image **130** to
16 produce an image dipole list **1300**. In a preferred embodiment, the feature detection module **200**
17 is identical to that used by training module **110**, and is controlled by the same parameter settings
18 stored in pattern parameters **220**, and is further described in conjunction with Fig. 26. In other
19 embodiments, different methods or different parameters setting are used as appropriate for a
20 specific application.

1 At least one attraction module **1350** uses pattern parameters **220**, field dipole set **230**, field **240**,
2 image dipole list **1300**, and the starting pose **132** and client map **131**, to refine the starting pose
3 **132** and produce the other outputs **136**, **138**, **160**, and **150**.

4 Figure 14 is a diagram that is used to derive the mathematical basis for a preferred embodiment
5 that uses a least-squares method to determine a pose that best accounts for the evidence of the
6 image dipoles at each attraction step. An image dipole, mapped by the current pose to field point
7 **1400** and with direction **1402**, is considered. A small section of image boundary **1404** is also
8 shown as an aid in understanding the diagram.

9 The field has force **f 1430** at mapped image dipole point **1400**, pointing to the nearest point **1435**
10 along pattern boundary **1410**. Note that the force **1430** is normal to the pattern boundary **1410** at
11 point **1435**, and the mapped image dipole direction **1402** is similar but not equal, modulo 180° ,
12 to that of the force.

13 The mapped image dipole point **1400** has position vector **p 1450** relative to the field origin **1420**.
14 The existence of an image dipole at field point **1400**, with force **1430** and mapped dipole
15 direction similar to the force direction (in a preferred embodiment, similar modulo 180°), is
16 taken as evidence that the current pose should be modified so that the image dipole is mapped so
17 as to lie somewhere along line **1440** tangent to pattern boundary **1410** at point **1435**. The dipole
18 provides no evidence as to position normal to the force, that is along tangent **1440**. The position
19 vector **p' 1470** defines a point on tangent **1440**, and the difference vector **p - p' 1460** indicates
20 how the mapped dipole position might move as a result of the force.

1 Suppose that $[\mathbf{C}, \mathbf{t}]$ is a six-degree-of-freedom coordinate transform that maps the current pose
 2 into a new, hopefully more accurate, pose. This transform is called the *motion* transform, because
 3 it tells how the image dipoles will move with respect to the field under the influence of the forces
 4 of the field. Here \mathbf{C} is a 2x2 matrix and \mathbf{t} is a translation vector. The evidence under
 5 consideration suggests that this transform should map \mathbf{p} to \mathbf{p}' :

$$6 \qquad \qquad \qquad \mathbf{p}' = \mathbf{C}\mathbf{p} + \mathbf{t}. \qquad \qquad \qquad (1)$$

7 Let \mathbf{I} be the identity matrix and define

$$8 \qquad \qquad \qquad f = |\mathbf{f}| \qquad \qquad \qquad (2)$$

$$9 \qquad \qquad \qquad \hat{\mathbf{f}} = \frac{\mathbf{f}}{|\mathbf{f}|} \qquad \qquad \qquad (3)$$

$$10 \qquad \qquad \qquad \dot{\mathbf{C}} = \mathbf{C} - \mathbf{I} \qquad \qquad \qquad (4)$$

11 From the diagram of figure 14 it can be seen that

$$12 \qquad \qquad \qquad f = (\mathbf{p}' - \mathbf{p}) \cdot \hat{\mathbf{f}} \qquad \qquad \qquad (5a)$$

$$13 \qquad \qquad \qquad = (\mathbf{C}\mathbf{p} + \mathbf{t} - \mathbf{p}) \cdot \hat{\mathbf{f}} \qquad \qquad \qquad (5b)$$

$$14 \qquad \qquad \qquad = [(\mathbf{C} - \mathbf{I})\mathbf{p} + \mathbf{t}] \cdot \hat{\mathbf{f}} \qquad \qquad \qquad (5c)$$

$$15 \qquad \qquad \qquad = (\dot{\mathbf{C}}\mathbf{p} + \mathbf{t}) \cdot \hat{\mathbf{f}} \qquad \qquad \qquad (5d)$$

1 Thus, given an image dipole that maps to field point \mathbf{p} with force $\mathbf{f} = f\hat{\mathbf{f}}$, we have one equation
 2 in the six unknowns $[\mathbf{C}, \mathbf{t}]$ that tells us how to map the current pose to get a new pose. With six
 3 dipoles we can solve for the six-degrees-of-freedom, but in practice the evidence obtained from
 4 only six dipoles is generally not sufficient to get an accurate or even meaningful solution. In
 5 practice we use many dipoles, typically anywhere from a few dozen to a few thousand, and some
 6 method for solving an over-determined set of equations.

7 In a preferred embodiment, a least-squares method is used. An error term for the i^{th} dipole can be
 8 defined as

$$9 \quad e_i = (\dot{\mathbf{C}}\mathbf{p}_i + \mathbf{t}) \cdot \hat{\mathbf{f}}_i - f_i \quad (6)$$

10 With this definition a least-squares problem can be set up and solved by methods well-known in
 11 the art. If weights w_i are determined for each dipole, we can write the sum squared error as

$$12 \quad E = \sum_i w_i \left[(\dot{\mathbf{C}}\mathbf{p}_i + \mathbf{t}) \cdot \hat{\mathbf{f}}_i - f_i \right]^2 \quad (7)$$

13 In practice it is usually desirable to solve for fewer than six-degrees-of-freedom. Some patterns
 14 would result in a singular or unstable solutions if certain degrees of freedom are included. For
 15 example, circles cannot be solved for orientation and corners cannot be solved for size. In other
 16 cases, a solution would be possible, but some degrees of freedom, particularly aspect ratio and
 17 skew, are known not to vary and might cause problems if included. Perhaps the most serious
 18 such problem is that unreliable evidence, present to some degree in all images, will have a more
 19 serious effect when more degrees of freedom are allowed to vary. Another problem is that
 20 somewhat more computation is needed to solve for the additional degrees of freedom.

1 In a preferred embodiment the least-squares problem is set up in 4 degrees of freedom
2 corresponding to x translation, y translation, orientation, and size. Sums needed for a least-
3 squares solution in the 4 degrees of freedom are computed, and pattern parameters **220** specify
4 which of the degrees of freedom will be solved for.

5 In an orthonormal coordinate system we can constrain the matrix $\dot{\mathbf{C}}$ to orientation and size
6 variation by writing it as

$$7 \quad \dot{\mathbf{C}} = \begin{pmatrix} p & q \\ -q & p \end{pmatrix} \quad (8a)$$

$$8 \quad = p\mathbf{I} + q \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad (8b)$$

9 In practical applications, however, the images themselves are almost never orthonormal. CCD
10 cameras, for example, typically have pixels that are non-square by a percent or so. For line scan
11 cameras, the angle between the coordinate axes depends on mechanical alignment and so the
12 coordinate axes may not be perfectly orthogonal. The variations from square are small, but easily
13 detectable given the accuracy that can be achieved with the invention. Furthermore, it is
14 sometimes useful to have a significantly non-orthonormal field. For example, a field generated
15 from a square pattern can be used to localize and inspect a rectangular or even parallelogram-
16 shaped instance of the pattern by using an appropriate starting pose.

17 In these cases we generally want the orientation degree of freedom defined by an orthonormal,
18 real-world coordinate system rather than image or field coordinates. We re-write $\dot{\mathbf{C}}$ as

$$\dot{\mathbf{C}} = p\mathbf{I} + q\mathbf{N} \quad (9)$$

where the elements of matrix \mathbf{N} are the components of the *normal tensor* in the field coordinate system. The normal tensor is a mixed 2nd-rank tensor, a vector-valued function of vectors that, informally, tells how to rotate a vector 90°. In an orthonormal coordinate system, of course, the components of the normal tensor are

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

The components of the normal tensor are computed from the current pose and from a coordinate transform called the *client map* that transforms points in an orthonormal but otherwise arbitrary reference coordinate system to points in the run-time image.

We can now re-write equation 7, the sum squared error, as

$$E = \sum_i w_i \left[[(p\mathbf{I} + q\mathbf{N})\mathbf{p}_i + \mathbf{t}] \bullet \hat{\mathbf{f}}_i - f_i \right]^2 \quad (10a)$$

$$= \sum_i w_i \left[p\mathbf{p}_i \bullet \hat{\mathbf{f}}_i + q\mathbf{N}\mathbf{p}_i \bullet \hat{\mathbf{f}}_i + \mathbf{t} \bullet \hat{\mathbf{f}}_i - f_i \right]^2 \quad (10b)$$

Now we can substitute

$$r_i = \mathbf{p}_i \bullet \hat{\mathbf{f}}_i \quad (11)$$

$$s_i = \mathbf{N}\mathbf{p}_i \bullet \hat{\mathbf{f}}_i \quad (12)$$

$$\mathbf{t} = \begin{pmatrix} x \\ y \end{pmatrix} \quad (13)$$

$$\hat{\mathbf{f}}_i = \begin{pmatrix} u_i \\ v_i \end{pmatrix} \quad (14)$$

into equation 10b and finally we have

$$E = \sum_i w_i [(xu_i + yv_i + pr_i + qs_i - f_i)]^2 \quad (15)$$

A least-squares problem based on equation 15 is easy to set up and solve for x , y , p , and q by well-known methods. The desired motion transform $[\mathbf{C}, \mathbf{t}]$ is obtained from said solution using equations 4, 9, and 13. The current pose is composed with the motion transform to obtain the new pose.

Figure 15 is a block diagram of the attraction module **1350** for a preferred embodiment based on a least-squares method of best accounting for the evidence of the image dipoles. In addition, to further clarify a preferred sequence of operation of the modules of Fig. 15, a flow chart is provided in Figs. 27A and 27B. Steps of the flow chart include reference numbers from Fig. 15 in parentheses to help cross-correlate the figures. A current pose **1500** is initially set to the start pose 2702 and updated at the end of each attraction step 2724. After the sum module 1535 is initialized to zero 2704, the Normal tensor computation module **1510** uses the current pose and client map to compute the normal tensor \mathbf{N} 2706 for the current attraction step.

Each image dipole **1515** of the image dipole list **1300** (figure 13) is processed 2708. The position and direction of dipole **1515** are mapped 2710 from image coordinates to field coordinates by

1 map module **1520**, using the current pose **1500**. The mapped position is used by field module
2 **1525** to determine the force, flags **430**, and index of nearest field dipole 2712. The force, flags,
3 and index are stored in the image dipole **1515** for later use. The normal tensor, force, and image
4 dipole position in field coordinates are used by a rotate module **1530** to obtain the dipole's
5 position in force coordinates (r, s) 2714 as specified by equations 11 and 12.

6 An evaluate module **1545** examines the force, flags, image dipole direction in field coordinates,
7 and dipole gradient magnitude and computes a weight for attraction (localization) purposes and
8 evaluation and clutter values for inspection purposes 2714. In some embodiments, the evaluate
9 module **1545** also considers the rms error from the previous attraction step in determining the
10 weight and evaluation. The evaluation and clutter values are stored in the image dipole **1515** for
11 later use.

12 A sum module **1535** uses the force, dipole position in force coordinates, and weight to compute
13 sums needed for the least-squares solution 2716. If there are no more dipoles 2718, a solve
14 module **1540** uses the sums and the normal tensor to solve for the motion transform and compute
15 the rms error 2720. A compose module **1505** composes the current pose with the motion
16 transform to produce a new pose 2722, which will be the current pose for the next attraction step,
17 or the final pose if this is the last attraction step 2726.

18 In a preferred embodiment where inspection is being performed, at the end of the last attraction
19 step, the field dipole evaluation module 1550 evaluates the image dipole list 2728, which has
20 now been evaluated by evaluate module **1545**, and the field dipole set **230** stored in the pattern
21 **120**, and produces an evaluated field dipole list, coverage rating, and clutter rating 2730.

1 Figure 16 gives details for the map module **1520** of figure 15. Inputs are image dipole position in
2 image coordinates **1600**, image dipole direction with respect to image coordinates **1610**, and the
3 current pose **1620**. One output is the dipole position in field coordinates **1630**, computed as
4 shown.

5 The other output is the dipole direction with respect to field coordinates **1640**, computed as
6 shown. The formula for output dir(field) **1640** effectively does the following, reading the vector
7 and matrix operations right to left:

- 8 • Construct a unit vector in the dipole direction, with respect to image coordinates,
9 by computing the cosine and sine of the angle θ_d .
- 10 • Rotate the unit vector 90° to get a direction along the boundary that contains the
11 dipole.
- 12 • Map the rotated unit vector to field coordinates using C_p to get a boundary
13 direction in field coordinates.
- 14 • Rotate the mapped rotated unit vector -90° to get a direction normal to the
15 boundary in field coordinates.
- 16 • If the determinant of the pose matrix is negative, the transform changes the left- or
17 right-handedness of the coordinate system, so rotate the vector 180° because the -90°
18 of the previous step should have been $+90^\circ$.

1 • Compute the angle of the resulting vector using the well-known version of the
2 arctangent function of two arguments whose result is in the range 0° to 360° .

3 Note as shown in output dir(field) **1640** that these calculations can be simplified considerably. In
4 a preferred embodiment, the simplified formula is used at the beginning of each attraction step to
5 compute a 256-element lookup table, indexed by an 8-bit binary angle, for use by the map block
6 **1520**. This allows the direction mapping operation to be executed at high speed for each dipole.

7 When computing the lookup table, the symmetry of the formula requires us to compute only 128
8 elements of the table; the other elements are the negative of the computed ones. As a further
9 improvement in computation time, 64 even-indexed elements are computed, and the odd-indexed
10 elements are determined by interpolation from the even-indexed elements. Thus the formula need
11 only be applied 64 times. The arctangent function is computed using the well-known CORDIC
12 method.

13 Note that in computing output dir(field) **1640** we map the boundary direction instead of the
14 dipole direction. This is because, in the embodiment described herein, directions are determined
15 by a gradient estimation method **330** and Cartesian to polar conversion method **340** that assumes
16 square pixels. This is not a problem except when mapping directions between non-orthonormal
17 coordinate systems. In that case, the boundary direction must be used.

18 Figure 17 shows a block diagram of the field module **1525** of figure 15, and a corresponding
19 geometric diagram that illustrates the computation being performed. Image dipole position in
20 field coordinates **1754** is input to the field block 1525. The coordinates **1754** fall within field grid
21 cell **1750**.

1 The coordinates are rounded to integer field grid position **1758** by integer rounding module **1700**.
2 The integer field grid coordinates **1758** are used by address generation module **1704** to produce a
3 memory address used to look up field element **1708**, corresponding to grid cell **1750**. In the
4 embodiment shown, the index of the nearest field dipole is stored with the other field
5 information, but in some embodiments, as described herein, the index is kept in a separate array.

6 The force direction θ_f , flags, and index obtained from field element **1708** are direct outputs of
7 field module **1525**, but the force magnitude **1774** is interpolated so that the force vector is a
8 reasonably smooth function of real-valued position within the field.

9 Force interpolation is based on the assumption that the force stored in field element **1708**,
10 corresponding to integer grid position **1758**, points to an approximately straight-line section of
11 pattern boundary **1782**. This is a fast and accurate interpolation we can use with the information
12 available. A more compute intensive interpolation could use neighboring field elements as well.

13 To interpolate force magnitude, the integer position **1758** is subtracted by module **1712** from the
14 real-valued position **1754** to produce sub-grid position vector **1762**. A unit vector **1766** in the
15 force direction is constructed by cosine/sine module **1716**, implemented as a lookup table in a
16 preferred embodiment. The dot product **1770** of sub-grid position vector **1762** and unit vector
17 **1766** is computed by dot product module **1720**. The dot product **1770** is subtracted from force
18 magnitude **1774** by the subtraction module **1724** to produce interpolated force magnitude **1778**.

19 The interpolated force magnitude **1778**, unit vector in the force direction **1766**, and force
20 direction angle stored in field element **1708**, are collected in output module **1728** and become
21 part of the force vector **1526** produced by field module **1525**.

1 In another embodiment, not shown, at least one force vector is stored in each field element,
2 pointing to the nearest points along at least one pattern boundary. The field module 1525
3 examines image dipole direction in addition to position, and uses the stored force vector that is
4 closest to the dipole direction for interpolation and output to subsequent steps.

5 Figure 18 gives details for the rotate module **1530** of figure 15. Inputs are the normal tensor
6 **1800**, force **1810**, and image dipole position in field coordinates **1820**. Output is image dipole
7 position in force coordinates $\text{pos}(\text{force}) **1830**, computed as shown, and as described above by
8 equations 11 and 12.$

9 Figure 19 shows various preferred fuzzy logic processing modules that are used in evaluate
10 module **1545** of a preferred embodiment illustrated in Fig. 20.

11 Figure 19a shows a *fuzzy greater than* module, which takes a real-valued input **1900** and a fuzzy
12 threshold **1904**, and produces a fuzzy logic value **1912**. The fuzzy threshold **1904** is an ordered
13 pair that specifies points along the x axis of graph **1908**. The graph **1908** shows the fuzzy logic
14 output **1912** as a function of input **1900**. As can be seen, the fuzzy logic value falls within the
15 range 0.0 to 1.0, inclusive.

16 Figure 19b shows a *fuzzy less than* module, which takes a real-valued input **1930** and a fuzzy
17 threshold **1934**, and produces a fuzzy logic value **1942**. The fuzzy threshold **1934** is an ordered
18 pair that specifies points along the x axis of graph **1938**. The graph **1938** shows the fuzzy logic
19 output **1942** as a function of input **1930**. As can be seen, the fuzzy logic value falls within the
20 range 0.0 to 1.0, inclusive.

1 Figure 19c shows a *fuzzy not* module. Fuzzy logic value input **1960** is inverted by subtracting it
 2 from 1 to produce fuzzy logic value output **1964**.

3 Figure 20 is a block diagram of a preferred embodiment of the evaluate module **1545** of figure
 4 15. Figure 20a shows the portion responsible for computing the *weight* and *eval* values, and
 5 figure 20b shows the portion responsible for computing the *clutter* value.

6 Referring to figure 20a, the computation of *weight* and *eval* is based on the force magnitude '*f*', a
 7 comparison of the force direction θ_f , and image dipole direction $\text{dir}(\text{field})$, and the image dipole's
 8 gradient magnitude '*mag*'. For each of these three factors in the evaluation, a fuzzy logic value is
 9 produced by fuzzy logic modules **2004**, **2040**, **2064**, respectively, that indicates confidence in the
 10 reliability of the evidence provided by the image dipole being evaluated. The three fuzzy
 11 confidence factors so-produced are combined into a single confidence score by a combination
 12 module **2080** in the range 0.0 to 1.0. The *weight* and *eval* outputs are obtained by using the eval
 13 code **440** (figure 4) to select either the confidence score or the value 0.0.

14 Absolute value module **2002** computes the length of the force vector from force magnitude *f*,
 15 which in the preferred embodiment being described may be negative if the force and gradient
 16 directions differ. Fuzzy less than module **2004** compares the force length to a field strength
 17 threshold **2000**, to produce a strength confidence factor that indicates high confidence for force
 18 lengths "below" the field strength threshold.

19 The field strength threshold **2000** is set based on pattern parameters **220** for the first attraction
 20 step. In a preferred embodiment, the first attraction step uses field strength threshold values $t_{\text{zero}} =$
 21 2.0 field grid units, and $t_{\text{one}} = 3.0$ field grid units.

1 In the embodiment shown in figure 20, the field strength threshold 2000 is modified after each
2 attraction step based on the rms error from the previous step. The modification is accomplished
3 by addition module **2012**, which adds the rms error to both the t_{zero} and t_{one} components of a field
4 strength margin parameter **2008** to produce the new field strength threshold 2000. As a result, the
5 field strength threshold 2000 is matched to how well the particular run-time image being
6 analyzed fits the stored pattern at each attraction step.

7 The method of adjusting the field strength threshold based on the rms error of the previous step is
8 effective in some applications, but in other cases it has been observed to result in some
9 oscillation of the attraction rather than convergence on one solution. In a preferred embodiment,
10 not shown, the field strength threshold is reduced in equal steps after each attraction step. Thus,
11 as the attraction converges to a solution, image dipoles must be closer to pattern boundaries to be
12 given high confidence.

13 The image dipole direction $dir(field)$ is compared with the pattern boundary gradient direction by
14 subtract module **2024**. Recall that in the embodiment being described, the “force” direction θ_f
15 reported by the field is actually boundary gradient direction, which is the same as or opposite of
16 the true force direction. If pattern parameters **220** and polarity code **460** of flags **430** specify that
17 gradient polarity is to be ignored, the angle difference from subtract module **2024** is constrained
18 to the range -90° to $+90^\circ$ by mod 180° module **2028**; otherwise, the angle θ_f is passed
19 unmodified. The magnitude of the resulting angle difference is determined by absolute value
20 module **2032**.

1 The angle difference magnitude is compared to one of two fuzzy thresholds by fuzzy less than
2 module **2040** to produce a direction confidence factor. If corner code **450** of flags **430** indicates
3 “no corner”, the field direction threshold **2044** is chosen by selection module **2036**. If the corner
4 code indicates “is corner”, the field corner threshold **2048** is chosen. In a preferred embodiment,
5 the field direction threshold **2044** has values $t_{zero} = 11.25^\circ$ and $t_{one} = 22.5^\circ$, and the field corner
6 threshold **2048** has values $t_{zero} = 39.375^\circ$ and $t_{one} = 50.625^\circ$, reflecting the fact that a wider range
7 of image dipole directions can reasonably correspond to a pattern boundary corner. In an
8 alternate embodiment, a real-valued measure of curvature can be used instead of the binary “is
9 corner” code, with multiple values of the field direction threshold possible.

10 The image dipole’s gradient magnitude ‘mag’ is compared to a fuzzy magnitude threshold **2060**
11 by fuzzy greater than module **2064** to produce a magnitude confidence factor. The magnitude
12 threshold is intended to throw out very weak dipoles that are likely due to image noise or other
13 artifacts, but the use of a fuzzy threshold gives more stable results than the more traditional hard
14 threshold. In a preferred embodiment, the magnitude threshold **2060** uses the same value for t_{zero}
15 as the noise threshold chosen for the peak detector **350**, and uses a value of t_{one} equal to twice the
16 value of t_{zero} .

17 The strength, direction, and magnitude confidence factors are combined by multiply module
18 **2080** to produce an overall confidence score in the range 0 to 1. Based on eval code **440** of flags
19 **430**, the selection module **2084** chooses a value for *weight* and the selection module **2088**
20 chooses a value for *eval*. If the eval code is “attract”, the confidence score is chosen for the
21 weight; otherwise the constant 0 is chosen so that the dipole is ignored for localization purposes.
22 If the eval code is “attract” or “evaluate only”, the confidence score is chosen for *eval*; otherwise

1 the constant 0 is chosen to indicate that the dipole does not correspond to any portion of the
2 pattern.

3 Figure 20b shows a preferred embodiment for the calculation of the *clutter* value. The direction
4 confidence factor produced by fuzzy less than module 2040 is inverted by fuzzy not element
5 2042. The image dipole's gradient magnitude is compared to a fuzzy clutter threshold 2070 by
6 fuzzy greater than module 2074 to produce a clutter confidence factor 2075. The clutter
7 confidence factor 2075 is multiplied by the inverted direction confidence factor by multiplier
8 2090 to produce a tentative clutter value 2091. If the eval code 440 is anything but "don't care",
9 selection module 2092 chooses this tentative clutter value 2091 as the dipole's *clutter* value;
10 otherwise the constant 0 is chosen.

11 If the eval code 440 is "expect blank", the force magnitude 'mag' is meaningless, but in a
12 preferred embodiment, the force direction θ_f encodes the gradient direction from the training
13 image as described above in conjunction with figure 5. In this case, the computation of clutter
14 uses this direction as it would the force direction θ_f . This mode of operation is appropriate when
15 it is desirable to minimize false alarms. Alternatively, if it is appropriate to minimize the chances
16 of missing clutter, one can consider only the clutter confidence factor 275 when the eval code is
17 "expect blank".

18 In a preferred embodiment, the clutter threshold 2070 has values of t_{zero} and t_{one} , each value being
19 equal to 1.5 times the t_{zero} and t_{one} values used for the magnitude threshold 2060.

20 In a preferred embodiment, the magnitude confidence factors and clutter confidence factors for
21 all of the image dipoles are computed once and stored in the image dipole list 1300, rather than

1 being recomputed for each attract step. This can be done because these confidence factors are
2 independent of the current pose **1500**.

3 Figure 21 is a block diagram of sums module **1535** of figure 15. This module accumulates the
4 weighted sums needed for the solution of the least-squares problem of equation 15. Five multiply
5 modules **2100** perform the weighting. Fifteen multiply-accumulate modules **2130** and one
6 accumulate module **2160** compute and store the sums needed. The sixteen accumulators are set
7 to zero at the beginning of each attraction step. This four degree-of-freedom case is an
8 exemplary embodiment, other numbers of degrees of freedom being possible.

9 Figures 22 a-d give details of the solve module **1540** of figure 15, which produces the motion
10 transform and the rms error value. The formulas shown are based on the solution of the least-
11 squares problem of equation 15. Pattern parameters **220** specify which degrees of freedom are to
12 be determined.

13 Figure 22a shows the solution for the 2 translation degrees of freedom only — size and
14 orientation are as specified in the start pose. Figure 22b shows the solution for translation and
15 orientation. This preferred solution is based on an approximation that assumes a small angle of
16 rotation. If the assumption is violated, some size variation will be introduced. Figure 22c shows
17 the solution for translation and size, holding orientation fixed, and figure 22d shows the solution
18 for all 4 degrees of freedom.

19 Figure 23 gives details of the compose module **1505** of figure 15, which composes the current
20 pose **2300** with the motion transform **2330** computed by the solve module **1540** to produce the
21 new pose **2360**.

1 Figure 24 gives details of the normal tensor module **1510** of figure 15, which computes the
2 normal tensor **2460** from the current pose **2400** and the client map **2430**.

3 Figure 25 shows an example of field dipole evaluation performed as part of field dipole
4 evaluation module **1550** of figure 15. In the example, a first image dipole **2500**, second image
5 dipole **2510**, and third image dipole **2520** have received evaluations 0.85, 0.93, and 0.88
6 respectively. Four field dipoles labeled **2540**, **2550**, **2560**, and **2570** lie along a chain as
7 determined by connect step **254** during training module **110**. The chain is defined by the left links
8 **2580** and right links **2585**.

9 For image dipole **2500**, an index **2505** was determined by field module **1525** to identify the
10 nearest field dipole **2540**. The evaluation 0.85 is transferred from image dipole **2500** to the
11 “eval1” slot of field dipole **2540**.

12 No image dipole identified field dipole **2550** as nearest, so its “eval1” slot holds its initial value
13 0.

14 For image dipole **2510**, an index **2515** was determined to identify the nearest field dipole **2560**.
15 For image dipole **2520**, the same index **2515** was determined to identify the nearest field dipole
16 **2560**. The larger of image dipole **2510** evaluation 0.93 and image dipole **2520** evaluation 0.88 is
17 transferred to the “eval1” slot of field dipole **2560**.

18 Field dipole **2570** has evaluation 0.90 transferred from some image dipole not shown.

19 To fill in the gap at field dipole **2550**, a dilation operation is performed, wherein all field dipoles
20 receive an evaluation equal to the maximum of their own evaluation and that of their left and

1 right neighbors. The dilated evaluations are shown in the “eval2” slot of each field dipole. Note
2 that it is not actually necessary to store both “eval1” and “eval2” values; figure 25 shows them
3 for clarity.

4 Once the field dipoles have been evaluated, the coverage value produced by field dipole
5 evaluation module **1550** is computed by averaging all of the field dipole evaluations.

6 In a preferred embodiment, the field dipoles are evaluated and coverage is computed only after
7 the last attraction step, and only if pattern inspection is desired.

8 Figure 26 shows how the invention can be operated in a multi-resolution mode designed to
9 increase the capture range without sacrificing accuracy. Two pattern training modules (not
10 shown) are run on a single training image, with different settings of low-pass filter module **310**
11 and image sub-sample module **320**. In a first setting designed to attenuate fine detail, a low
12 resolution pattern **2600** is generated. In a second setting designed to pass fine detail, a high
13 resolution pattern **2610** is generated.

14 A low resolution run-time module **2620** uses the low resolution pattern **2600**, and a start pose and
15 client map, to analyze run-time image **130** to produce a low resolution pose that is much more
16 accurate than the start pose but not as accurate as can be achieved at higher resolution. A high
17 resolution run-time module **2630** uses the high resolution pattern **2610**, the low resolution pose
18 as a start pose, and the same client map, to analyze run-time image **130** to produce the final pose,
19 rms error, coverage, and evaluated dipole lists.

20 The multi-resolution mode is supervised by overall control module **2640**, as illustrated by the
21 flow chart in Fig. 28. As part of its operation 2800, the low resolution rms error, coverage, and

1 clutter values are examined, and if 2802 they do not indicate a reasonable match between image
2 and stored pattern, the operation is aborted 2803 without attempting to run the high resolution
3 module. If the low resolution module produces a good match, high resolution module is run
4 2804. If the high resolution module does not produce good results 2806, it usually means that the
5 image is out of focus, and the user is so-warned. In some embodiments, when this happens, the
6 low resolution results are used instead of the high resolution results 2808. If the results of the
7 high resolution module are acceptable, the results of the high resolution module are provided to
8 the user 2810 for interpretation, or further processing, according to the particular application.

9 In a preferred embodiment, an overall match score is computed for each resolution step that is
10 equal to the coverage value minus half the clutter value. The low resolution results are used
11 instead of the high resolution results if the high resolution match score is less than some fraction
12 of the low resolution match score. In a preferred embodiment, the fraction used is 0.9.

13 In a preferred embodiment, the methods of co-pending US patent application Serial No.
14 _____, filed 11/14/97, entitled "Efficient, Flexible Digital Filtering", and co-pending US
15 patent application Serial No. 08/727,721, filed 10/07/96, entitled "Fast, Inexpensive, Subpixel
16 Edge Detection" are used for feature extraction, Cognex Corporation's PatQuick™ tool is used to
17 determine the starting pose, and the multi-resolution style of figure 26 is used. The following
18 parameter settings are used for feature extraction by default. Many other strategies can be devised
19 to suit specific applications.

20 For training the low resolution pattern 2600, and corresponding run-time module 2620, the image
21 is sub-sampled by sub-sampler 320 by an equal amount in x and y given by the formula

1
$$\text{floor}\left(\sqrt{\frac{\sqrt{wh}}{8}}\right)$$

2 where w and h are the width and height, respectively, of the pattern **100** in pixels and the *floor*
3 function gives the largest integer that is less than or equal to its argument. Note that sub-sampling
4 by n means taking every n^{th} pixel. The low-pass filter **310** uses a filter size parameter (“ s ” in co-
5 pending US patent application Serial No. _____, filed 11/14/97, entitled “Efficient,
6 Flexible Digital Filtering”) equal to one less than the computed sub-sample amount. The
7 Cartesian to polar conversion module **340** multiplies the gradient magnitude values by 2.0 to
8 improve precision at the low end, where most gradient values lie.

9 For training the high resolution pattern **2610**, and corresponding run-time module **2630**, the low-
10 pass filter **310** and the sub-sampler **320** are set to pass the source image **300** unmodified.

11 As part of its operation, the PatQuick™ tool reports a “contrast” value in gray levels that is the
12 median gradient magnitude of the pixels in the image on which it is run that correspond to the
13 trained pattern. In a preferred embodiment, this contrast value is used to set the default noise
14 threshold for the peak detector **350**. Many other schemes for setting noise thresholds are known
15 in the art that can be used to achieve equivalent results.

16 In said preferred embodiment, PatQuick™ is run on the training image **100** and the contrast value
17 reported by the tool is saved as part of the pattern parameters **220**. For training the low resolution
18 pattern **2600**, the peak detection module **350** uses a noise threshold equal to 10 gray levels. For
19 training the high resolution pattern **2610**, the peak detection module **350** uses a noise threshold
20 equal to one-quarter of said saved contrast.

1 For the run-time image **130**, when the PatQuick™ tool is used to determine the starting pose the
2 contrast value it reports is examined. For both the low resolution run-time module **2620**, and the
3 high resolution run-time module **2630**, the peak detection module **350** uses a noise threshold
4 equal to that used for the corresponding pattern **2600** or **2610**, but in each case multiplied by the
5 ratio of run-time contrast to the saved train-time contrast.

6 The preferred embodiments described herein use a six-degree-of-freedom coordinate transform to
7 represent the mapping between points in the image and points in the pattern (i.e. the pose), and a
8 least-squares fitting to determine how to use the information provided by the field to modify a
9 given pose so as to produce a new pose that represents a better correspondence between image
10 and pattern features. Many other arrangements can be devised by those of ordinary skill in the art
11 for achieving similar results within the scope of the invention. These other arrangements may
12 have advantages in certain specific applications.

13 For example, the six degree of freedom coordinate transform can be replaced with other analytic
14 models of the mapping between points in the image and points in the pattern. One useful such
15 model is the well-known perspective transform. Another useful model is one that corrects for
16 lens distortions, such as that produced by so-called “fisheye” lenses. In these cases a different
17 least squares solution would be used, and appropriate changes would be made to the pose
18 element **1500**, the compose module **1505**, the normal tensor module **1510**, the map module **1520**,
19 the rotate module **1530**, the sums module **1535**, and the solve module **1540**. The image dipole
20 **1515**, field module **1525**, evaluate module **1545**, and field dipole evaluation module **1550** need
21 not change.

1 In other arrangements, the least squares method can be replaced with other well-known methods
2 for fitting data. In such arrangements, appropriate changes might be made to the rotate module
3 **1530**, the sums module **1535**, and the solve module **1540**. Alternatively, one or more of these
4 modules might be replaced by different modules that are required for the fitting method to be
5 used.

6 In still other arrangements, a non-analytic mapping between points in the image and points in the
7 pattern, such as a 2-dimensional lookup table with interpolation, may be used. In such an
8 arrangement, the pose **1500** is a lookup table mapping image points to pattern points, and the
9 map module **1520** does the lookup and interpolation. The field module **1525** and evaluate module
10 **1545** can be used without modification. The compose module **1505**, normal tensor module **1510**,
11 rotate module **1530**, sums module **1535**, and solve module **1540** are not used. Instead, an
12 intermediate lookup table is produced as follows. For every image dipole **1515**, an entry is made
13 in the intermediate lookup table by adding the force vector obtained from the field module **1525**
14 to the mapped position from map module **1520**. Along with this field-corrected position, the
15 weight obtained from the evaluate module **1545** is also stored in the intermediate table entry.

16 The intermediate table thus produced may be sparse, in that many points will not have been filled
17 in, and it may have errors caused by the occasional unreliable image dipole. It can be used,
18 however, to produce a new pose **1500** by applying a *smoothness* constraint. For example, each
19 element of the new pose can be determined by a weighted mean or median of some neighborhood
20 of corresponding elements of the intermediate table. Other methods for using smoothness as a
21 constraint are well-known in the machine vision literature.

1 Other modifications and implementations will occur to those skilled in the art without departing
2 from the spirit and the scope of the invention as claimed. Accordingly, the above description is
3 not intended to limit the invention, except as indicated in the following claims.

4